

# Mathematical knowledge of first-year students in the light of Semiotic Representation Registers

# Conhecimento matemático dos alunos da primeira série à luz dos Registros de Representação Semiótica

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#### ABSTRACT

Many students face difficulties in completing high school satisfactorily due to lack of understanding of essential mathematical concepts. These difficulties can result in low grades, failure and even dropout. The Maceió campus, of the Federal Institute of Alagoas, receives more than 500 students per year, many of whom have difficulties in the subject of mathematics. To address this issue, we carried out a survey that uses the Theory of Semiotic Representation Registers to identify the deficiencies of first-year high school students on the Maceió campus. To achieve this objective, we first conducted a systematic literature review that investigated how the Theory of Semiotic Representation Registers has been used to analyze high school students' mathematical knowledge. After that, a five-question test was applied to 56 students from three of the seven courses. The analysis of the responses showed us that the students had great difficulty in converting the text in their mother tongue to algebraic language and in the question of numerical expression, which involved the conversion to only one record.

#### RESUMO

Muitos estudantes enfrentam dificuldades em cursar o Ensino Médio de forma satisfatória devido à falta de compreensão de conceitos matemáticos essenciais. Essas dificuldades podem resultar em notas baixas, reprovação e até mesmo evasão. O campus Maceió, do Instituto Federal de Alagoas, recebe mais de 500 estudantes por ano, muitos dos quais têm dificuldades na disciplina de matemática. Para abordar essa questão, realizamos uma pesquisa que utiliza a Teoria dos Registros de Representação Semiótica para identificar as deficiências dos alunos da primeira série do Ensino Médio no campus Maceió. Para atingirmos este objetivo, primeiramente, conduzimos uma revisão sistemática da literatura que investigou como a Teoria dos Registros de Representação Semiótica tem sido usada para analisar o conhecimento matemático dos alunos do Ensino Médio. Após isso, um teste de cinco questões foi aplicado em 56 estudantes de três dos sete cursos. A análise das respostas nos apresentou que os estudantes apresentam grande dificuldade na conversão do texto em língua materna para a linguagem algébrica e na questão de expressão numérica que envolvia a conversão para somente um registro.

#### ARTICLE INFORMATION

Histórico do Artigo: Submitted: 08/09/2023 Approved: 11/12/2023 Published: 19/12/2023



#### Keywords:

mathematical learning, theory of semiotic representation registers, mathematical concepts, elementary school 2.

#### Palavras-Chave:

aprendizagem matemática, teoria dos registros de representação semiótica, conceitos matemáticos, ensino fundamental anos finais.

## Introduction

When they reach high school, many students face difficulties in the subject of Mathematics because they have gaps in the learning of mathematical content in Elementary School. The SAEB test (INEP, 2021), for example, exposes this difficulty. The result of the last test carried out with students of the 9th year of Elementary School for the subject of Mathematics had a national average of 256 points. In the state of Alagoas, the average was 247.8 points, leaving the state in 17th place among 27 states.

These difficulties lead to low grades, recoveries and, in many cases, dropouts. According to data from the School Census (INEP, 2022), in 2019, the failure rate (failure + dropout) of students in the first year of high school in Brazil is 21.3% for public school students and, within the three years of high school, it is the highest rate.

There are also problems in the Federal Education Network. According to the data available on the Nilo Peçanha platform, the average dropout and retention during 2017 - 2021 is 40.51%. In other words, almost half of the students did not complete the course within the expected deadline or left the Institution.

Faced with this situation, several researchers have proposed to identify what these difficulties are and, based on them, propose alternatives so that these students can overcome them and attend a satisfactory high school (Costa et al., 2017; Netherlands et al., 2020; V. A. Oliveira et al., 2017; Ramos & Curi, 2014). In addition, the researches are based on theories of Mathematics Education so that they can find these difficulties or propose alternatives to solve them.

In this way, we developed a research whose objective was to identify the difficulties in mathematical contents of Elementary School - Final Years that accompany students entering High School at the Federal Institute of Alagoas - Maceió Campus. For this analysis, we chose the Theory of Registers of Semiotic Representation because we believe that it can help us to understand how students understand the mathematical objects that they studied in Elementary School and how they work with the treatments and conversions of these objects.

In the next section, we will bring what methodological paths we took to achieve the objective of this research. In the following section, we present a little about what the Theory of Registers of Semiotic Representation is. Then, we show the results found and, finally, we point out our final considerations.

### The Register Theory of Semiotic Representation

Raymond Duval is a French researcher who studies cognitive psychology and has developed an important theory for Mathematics Education. For the researcher, the importance and variety of language forms in mathematical activities was an area that caught his attention and made him study more deeply. For him, "the difficulties of comprehension in learning mathematics are not related to the concepts, but to the variety of semiotic representations used and the 'confusing' use of them" (Freitas and Rezende, 2013, p. 15). Since mathematical objects are abstract entities, the representations that are given to these objects have an immense importance for the learning of mathematics. According to Stormowski, Gravina and Lima (2013, p. 5), Duval considers that

It is from the coordination of several semiotic representations of the same object that one has access to the effective concept of the object considered, and that integral comprehension occurs with the coordination of registers that manifests itself with the speed and spontaneity of the cognitive activity of conversion. In other words, in order for learning in mathematics to occur, there is a need to coordinate several registers of semiotic representation.

For example, when we work with the mathematical object "rational numbers" we cannot teach only fractions, because it is present in decimal numbers, percentages, images of pizza slices, etc. In view of this, Duval (2012) says that, for mathematical activity, we must mobilize many registers of semiotic representation (figures, graphs, natural language, etc...) in the course of the same step, being able to choose one instead of the other. In other words, it is not enough to teach students only one representation of a certain object, the student must know how to work with several representations and know how to move between them.

In order to deal with these various types of records, Duval, in his theory, works with two processes: treatment and conversion. Duval (2009, p. 57) defines treatment as "the transformation of an internal representation into a representation register or a system". Conversion, on the other hand, is "an external transformation in relation to the registration of the starting representation" (Duval, 2009, p. 59).

For example, when we add 3/5+4/5 and find the result 7/5, we are doing a treatment because we have not yet left the representation of the system (fractions). But, we would be performing a conversion if we put the answer as 1.4, because we left the initial representation register (from fractions to decimals).

The great point of Duval's idea is that we cannot confuse the mathematical object with any of its representations. For this reason, the author states that we must teach at least two representations of the same object and that we can know how to perform the conversion between these representations. According to Duval and Moretti (2018), the main objective of mathematics teaching is to make students understand and use the specific way of thinking and working that is typical of mathematics. The spontaneous recognition of this object in different registers is a prerequisite for the student to be able to understand mathematics and apply it in real life situations. Thus, the theory of registers of semiotic representation presents itself as an important tool for the teaching and understanding of mathematics.

### Methodology

Exploratory in nature, this research first carried out a Systematic Review of the Literature following the steps proposed by Wohlin et al. (2012): planning the review (at this stage, the need for a review should be identified, the specification of the research guiding question and the creation of a review protocol); conducting the review, and; report of results.

We searched the portals Scielo, Scopus and Portal de Periódicos da CAPES, where, on November 19, 2022, the search string ("Registros de Representação") OR ("Registers of semiotic representations") OR ("Registers of semiotic representations") OR ("Registers of representation") OR ("Registers of semiotic representations") OR ("Registers of representation, we filter the works that were written in portuguese. Of the 144 studies found, 20 were duplicates and of the remaining 120, the following exclusion and inclusion criteria were used:

- CE1 Does not analyze students' prior knowledge;
- CE2 Does not present an abstract;
- CE3 Not in Portuguese;
- EC4 Not used in TRSS;
- CE5 Not for the subject of Mathematics;
- CI1 Alignment with the research methodology;
- CI2 Analyzes students' knowledge.

After this first stage, from the articles read for the literature review and other works on the theory that we will analyze the students' answers, we built a test containing five questions that addressed the contents: integers, rational numbers, area and perimeter and algebra. These contents were chosen based on the experience of the teachers in the classroom and the difficulties experienced with their students about these contents. Each question sought to analyze how the students use the conversion and treatment of the representation records that were addressed in each question.

The first question was adapted from Hillesheim and Moretti (2020) where it is expected that the student will be able to solve the problem about integers using any type of representation, whether discursive or non-discursive.

### Frame 1.

In the morning, thermometers in a city in northern Canada registered -1°C. By noon, the temperature had risen by 5°C. Determine the temperature of the city at this time of the measurement.

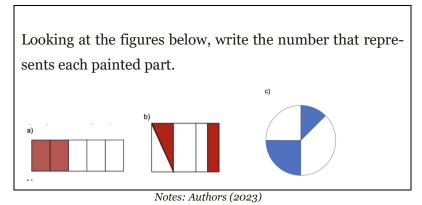
Notes: Authors (2023)

The second question addresses the content of the rational numbers in the geometric register and is asked to be converted to the numerical representation. Based on the examples

of Oliveira (2015) and Silva (2018), three figures are presented in the question:

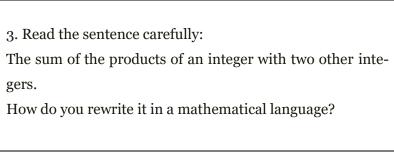
- the second divided the rectangle into three smaller ones where two are divided into different shapes, but with the same area;
- The third figure shows the circle sliced into four parts that do not have the same area.





The third question (Moretti et al., 2022) addresses the conversion between the plurifunctional discursive register and the monofunctional register. That is, students are expected to be able to write the sentence for the algebraic register.

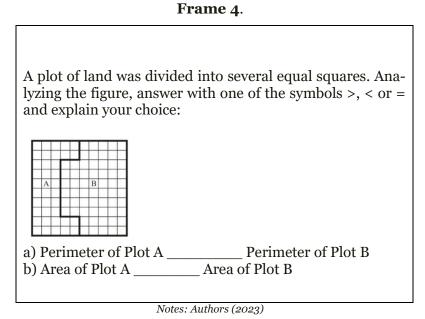




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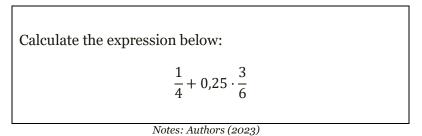
In the fourth question (Moretti et al., 2022), the subjects addressed are the area and perimeter where Duval (2012) calls perceptual apprehension in situations involving figures works. For the author, two types of seizure are highlighted: one immediate and the other controlled. Thus, the student is expected to move between the parts of the image in one dimension and in two dimensions. According to the author,

These two attitudes are usually in conflict because the figure shows objects that stand out independently of the statement and that the objects named in the statement of hypotheses are not necessarily those that appear spontaneously (Duval, 2012, p. 120).



The last question aims to analyze whether students can answer a numerical expression where it is necessary to convert to only one record. Because, according to the authors of the work, "the calculation algorithms change depending on the numbering system used, although from a mathematical point of view, the arithmetic operations are the same". (Duval & Morettti, 2018, p. 7).





The test was applied to 56 students (who did not have their personal data collected) distributed in three classes of technical courses integrated to high school: Electrotechnics (23), Roads (19) and Systems Development (14).

## Systematic Review of the Literature

Of the 120 studies analyzed, only 4 studies (all from the CAPES Journal Portal) targeted high school students. Regarding the contents analyzed, we observed in Table 1 that we had a study for quadratic equations, one for functions, one for a function of the second degree, and another for the fractions where it was analyzed in students of Elementary, Secondary, and Higher Education.

Authors	Content	Aspect of the theory used	
Siqueira and Bellemain (2011)	Quadratic equation	Conversion	
Paraol and Rodriguês <u>(2018)</u>	Fractions	Treatment and conversion	
Santana, Gualandi and Soares <u>(2019)</u>	Function of the second degree	Conversion	
Rezende, Lorenzoni and Souza <u>(2018)</u>	Function	Treatment and conversion	

Table 1.

Notes: Authors (2023)

Siqueira and Bellemain (2011) investigated the difficulties of students in the 3rd year of high school in the process of conversion between three types of representations in the algebraic register and the graphic register of the quadratic equation because, for the authors, this conversion is treated as trivial by the Brazilian education system even if, in reality, this is not so noticeable in the students.

The forms of representation in the algebraic register used were the factored -  $y=ax^2+bx+c$ , the canonical -  $y=a[(x+b/a)^2+(4ac-b^2)/4a]$  and the factored - y=a(x-x')(x-x''). For the authors,

Each form of the quadratic equation provides numerical information that relates to different information in its geometric representation, such as: the intersection with the abscissa axis, with the ordinate axis, the vertex coordinates, or even the orientation of the curve. (Siqueira & Bellemain, 2011, p. 6)

For the research, an activity was applied with 10 students of the 3rd year of High School of a federal public school in Recife. The questions contained in the activity dealt with conversion in both directions, that is, from the graphic register to the algebraic and from the algebraic to the graphic.

The results indicated that the students presented difficulties in the conversion between the registers, especially in the passage from geometric to algebraic representation. In addition, it was observed that many students limit themselves to using only one of the representations, not being able to relate the two in an integrated way. The authors highlight the importance of working with activities that promote the articulation between representations, enabling a deeper understanding of the concept of quadratic equations. The study contributes to the field of Mathematics Education by demonstrating how the theory of registers of semiotic representation can be applied in the teaching of quadratic equations, offering subsidies for the development of more effective pedagogical practices. As a suggestion, the study also presents solutions to each difficulty encountered, among them: a proposal of activities and situations that highlight the use of conversions, as well as more visual representations and teaching methods for better understanding.

Paraol and Rodriguês (2018) focus on the fractions that, according to the authors, there is a great difficulty in the teaching and learning process of this content due to the lack of understanding of the object itself. A diagnostic evaluation was applied to 23 students of Elementary School – Final, Middle and Higher Years with the aim of identifying how the students perform the treatment and conversion of fractional, decimal and figural representations.

The study presents a conclusion for each phase studied. In Elementary School, the great deficit is pointed out to the use of decimal numbers, in High School there is an improvement in the representation of fraction in decimal numbers, however, a difficulty in the representation of fraction in the figural register. As expected, Higher Education is the one with the lowest rate of errors, but they are similar errors, that is, especially in the decimal and figural representation of the mixed fraction.

The authors highlight the importance of the articulation between the different registers of mathematics for the success of learning, as highlighted by the theory. From the analysis of the data obtained, it is possible to infer that the teaching of fractions still needs more attention from teachers and researchers, since there seems to be no complete mastery of the records of this object, as well as of the conversions and treatments between fractions and their records.

The work of Santana, Gualandi and Soares (2019) investigated how students in the first year of Integrated High School use the different types of registers of semiotic representation in the teaching of quadratic functions and what these difficulties are. Through the analysis of the activities that were applied to the students, the study presents some possible reasons for the difficulty of transition between registers of a mathematical object, namely: difficulty in visualization; algebraic representation based on the graphic register (the most difficult way to understand quadratic function, according to students); and when points are highlighted on a graph and algebraic representation is requested.

Students also complain about a very shallow approach to Cartesian plans in previous years, also presenting another possible reason for the deficit. The authors point out that the results of the research may contribute to a possible change in the teaching practice of approaching mathematical contents, directing teaching to a broad teaching approach, in several different ways and representations.

Rezende, Lorenzoni and Souza (2018) examined how 33 students from a second-year high school class approach the different registers of semiotic representation of the mathematical object function using a mathematical modeling activity on charcoal production. The research was qualitative and used as data collection instruments the written records of the students, the logbooks of the researchers and the audio recordings. The results indicated that the students had difficulties in mobilizing more than one register of semiotic representation and in effecting transformations within the same register, which evidences an insufficient understanding of the concept of function. In addition, students tended to dissociate mathematics from the real world and had difficulties applying learned concepts in new contexts.

The researchers suggest that modeling activities associated with the registers of semiotic representation can contribute to the process of teaching and learning functions. These activities can provide the development of skills that allow the student to intervene in reality, build a more critical look at mathematics and construct mathematical concepts more efficiently. The mobilization of the different registers of semiotic representation can facilitate the construction of connections between the mathematical concept and reality, helping to overcome the tendency of students to dissociate mathematics from the real world and to transfer/apply content already studied to new contexts.

Thus, we found that the research focuses on the analysis of the students' knowledge in a given content, without considering the students' previous knowledge from the perspective of RRT. Thus, there is still a gap in research that proposes to analyze the mastery of students in the treatment and conversion of contents of Elementary School – Final Years, which are fundamental for good performance in High School.

## Analysis of test results

After the test was applied, the answers were corrected and three tables were obtained: how many students were correct, wrong and failed to answer question "x" (Table 2).

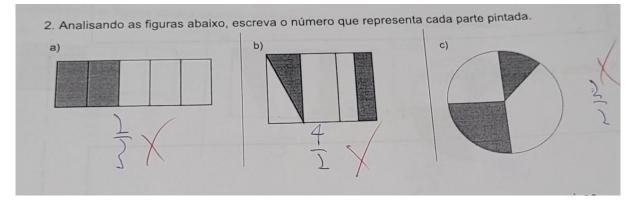
Question	Hits	%	Errors	%	Didn't respond	%
Q1	35	62,5%	17	30,4%	4	7,1%
Q2A	46	82,1%	10	17,9%	0	0,0%
Q2B	29	51,8%	27	48,2%	0	0,0%
Q2C	24	42,9%	32	57,1%	0	0,0%
Q3	2	3,6%	27	48,2%	27	48,2%
Q4A	17	30,4%	36	64,3%	3	5,4%
Q4B	37	66,1%	15	26,8%	4	7,1%
Q5	11	19,6%	29	51,8%	16	28,6%

Table 2.

Notes: Authors (2023)

When we analyzed the data from question 1 (Q1), we noticed that the use of these questions, i.e., their number of correct answers, was equivalent to 62.5%, demonstrating a good control of the whole numbers by the students. The types of representation presented by them were quite varied, so there was no predominance of a response in the written or numerical record. In the answers to question 2 (Q1A, Q2B and Q2C) it is evidenced that the students do not have difficulty with fractions in geometric representation, except when it does not have a division of equal areas, because question "a" had the highest rate of correct answers of all questions (82.15%), a scenario that changes in the other alternatives "b" and "c", where, respectively, they had a success rate of 51.8% and 42.85%. In Figure 1, we see a student who got the three letters wrong because he had not yet mastered the concept of fraction.





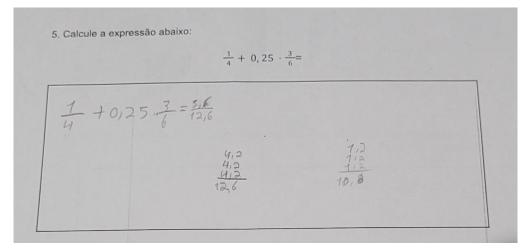
#### Notes: Authors (2023)

Question 3 (Q3) is the one with the lowest success rate, with only 2 correct answers, equivalent to 3.5% of the students, thus exposing an urgent concern with the knowledge of the conversion of the written record to the algebraic record of the students.

When developing the information obtained from question 4 (Q4A and Q4B), it is evident that there is an enormous difficulty with the concept of perimeter, considering that the rate of correct answers for alternative "a" is much lower than that of alternative "b", 30.35% for the first compared to 66% for the second.

The last one, question 5 (Q5), was another one with a very poor performance (19.65%), bringing up a great difficulty in transitioning between different records in the same question. Many students had the difficulty of deciding to which register the expression should be transformed. Other students did not even show this awareness, as we see in Figure 2.





Notes: Authors (2023)

# **Final Thoughts**

The objective of this study was to identify the difficulties in mathematical contents of Elementary School - Final Years that accompany students entering High School at the Federal Institute of Alagoas - Maceió Campus. For this, we used the Theory of Registers of Semiotic Representation to analyze the answers that the students made in a five-question test developed in this research.

With the systematic review we conducted, we realized the difficulty that students have in converting between different representations. In addition, it is emphasized the importance that the classes and activities that are created by the teachers are directed to these difficulties and that we do not work with the representations of the objects in isolation, but in such a way that the students can recognize the mathematical object in its different forms.

We can also see how other studies have used theory to analyze students' knowledge of mathematical concepts. This was very important for us to be able to develop our test, which was applied to students from the first grade classes of the Maceió Campus.

From the results we found, we are alerted to the large number of errors in the algebra question. Algebraic literacy for students who are entering high school is extremely important so that they can satisfactorily take the subject of Mathematics, in addition to others such as Chemistry and Physics. In addition, of the 56 answers, two of which were done correctly, half got it wrong and half didn't do it at all. This shows how important it is to work in this direction.

The last question, which dealt with a mathematical expression with different representations, also presented a high rate of errors and students who did not do so. In other words, one more point that we believe needs to be worked on with the students. In addition, in line with what we found in the literature review, specifically in the work of Paraol and Rodriguês (2018), it is still necessary to seek methodologies for teaching fractions and their different representations, since students had difficulties in questions related to this content.

We believe that this work does not stop at you. Due to the time and the fact that we had only one scholarship holder to administer the questionnaires, we were unable to visit more first-grade classrooms. A visit to more classrooms and more classes would give us a more accurate picture of what we have on campus. Besides that, this work is extremely important for the Campus and for the Learning Support program, as we will be able to have a more accurate picture of what are the greatest difficulties that our students carry with them since Elementary School.

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