

Error analysis in limit problems: error in conceptual understanding and mathematical procedure

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ABSTRACT

This study aimed to identify and analyze the conceptual understanding errors and mathematical procedural errors committed by Grade 11 STEM students in solving limit problems in calculus. Using a descriptive research design, data were collected from 25 students through five classroom activities aligned with the senior high school Basic Calculus curriculum. Newman's Error Analysis served as the primary analytical framework, and the results were interpreted through the OECD Six-Stage Problem-Solving Framework. The analysis revealed five recurring conceptual error themes: incomplete understanding of definitions, difficulty classifying and recognizing limit categories, challenges in handling complex problems, misinterpretation and misuse of mathematical symbols and notation, and confusion over terminology. Four procedural error themes were also identified: misapplication of mathematical rules, incomplete solution steps, inconsistent notation usage, and inappropriate strategy selection. These errors manifested at multiple stages of problem-solving, underscoring the need for targeted instructional interventions. The findings directly inform teaching strategies by highlighting specific areas where curriculum scaffolding, conceptual reinforcement, and metacognitive practices can meaningfully improve students' performance in calculus.

RESUMO

Este estudo examinou os erros comuns tanto na compreensão conceitual quanto no procedimento matemático na resolução de problemas envolvendo limites em cálculo, utilizando um delineamento de pesquisa descritiva. Os dados foram coletados de 25 estudantes do 11º ano da área STEM por meio de cinco atividades em sala de aula alinhadas ao currículo. A Análise de Erros de Newman foi utilizada para analisar os erros, sendo interpretada por meio do Modelo de Seis Estágios da OCDE. Os resultados revelaram cinco erros principais na compreensão conceitual: compreensão incompleta das definições, dificuldades em classificar e reconhecer diferentes categorias, dificuldade em lidar com problemas complexos, interpretação incorreta e uso inadequado de símbolos e notações, e confusão quanto à terminologia. Já os erros no procedimento matemático incluíram a aplicação incorreta de regras matemáticas, etapas incompletas, uso inconsistente de notações e escolha inadequada de estratégias. Esses erros surgem em várias etapas da resolução de problemas, como enfatizado no estudo, e reforçam a necessidade de desenvolver estratégias instrucionais. Além disso, o estudo promove a integração da análise de erros no ensino para aprimorar e desenvolver as habilidades matemáticas dos estudantes.

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Introduction

Mathematics is widely recognized as a discipline that fosters critical, analytical, and logical thinking, as well as problem-solving skills — competencies that the National Council of Teachers of Mathematics (NCTM, 2020) identifies as essential for 21st-century learners. Despite its importance, mathematics remains one of the most challenging subjects for students at all levels. Research consistently shows that many students struggle not only with computations but with grasping underlying mathematical concepts (Rahman et al., 2022).

Among the branches of mathematics, calculus stands out for its abstract nature and the depth of understanding it demands. Students frequently encounter difficulties related to limited conceptual knowledge, poor procedural application, complex formulas, and confusion in interpreting problem statements (Domondon, 2022). Keeping students meaningfully engaged in calculus requires instructional approaches that bridge the gap between abstract theory and practical problem-solving.

Within calculus, the concept of a limit is foundational — it underpins continuity, derivatives, and integrals. However, limits are particularly difficult to master because they require abstract thinking about the behavior of functions. When students form misconceptions about limits early on, these misunderstandings tend to cascade, impeding their grasp of more advanced calculus topics. Conceptual understanding of a limit, as a process describing the behavior of a function as its input approaches a particular value, must therefore be established clearly and precisely.

The challenges students face in calculus are well-documented across different educational contexts. Studies show that common errors include the incorrect application of rules (procedural errors), misunderstanding of fundamental concepts (conceptual errors), and incomplete or faulty recall of generalizations (Köğce, 2022). Related research in integral calculus similarly reveals errors tied to limited conceptual knowledge, incorrect procedures, and misinterpretation of graphical representations (Elguenyari et al., 2023). Conceptual misunderstandings and algebraic errors have been identified as the most frequent barriers to calculus comprehension (Smith, 2020), with students particularly struggling with the formal epsilon-delta definition of limits and the concept of infinity (Adams, 2012).

What is notably absent from much of the existing literature, however, is a detailed classification of errors specifically within limit problems at the senior high school level. Most prior studies address general misconceptions in calculus without unpacking the precise nature or origin of errors in limit-solving. Moreover, research has tended to examine conceptual and procedural errors in isolation. Yet mathematical procedure and conceptual understanding are deeply interconnected — a student's procedural mistake often signals an underlying conceptual gap, and vice versa (Schneider, 2015). Students also frequently select inappropriate strategies

when working on specific limit subtopics (Agustyani et al., 2019), further demonstrating the need for a more targeted analysis.

This study addresses that gap by providing an in-depth, dual analysis of both conceptual and procedural errors in solving limit problems. By categorizing errors into conceptual and procedural dimensions, teachers can gain clearer insight into the root causes of student difficulties — knowledge that is essential for designing effective remediation. According to Ashlock (2010), engaging students in reflecting on their own errors cultivates metacognitive awareness, enabling them to identify and self-correct misconceptions. Error analysis, integrated thoughtfully into instruction, thus becomes a powerful pedagogical tool.

Ultimately, understanding where and why students err in limit problems equips teachers to align their instructional strategies with students' actual learning needs. This study therefore serves as a practical resource for mathematics educators: it identifies specific error patterns, links them to stages of problem-solving, and offers evidence-based recommendations for instructional improvement — all in service of helping students build a stronger foundation for higher mathematics.

Specifically, this study examined and classified the errors in conceptual understanding and mathematical procedure among Grade 11 STEM students solving problems involving limits. It sought to identify recurring error patterns, pinpoint the stages at which errors occur — such as misunderstanding the formal definition of limits or applying incorrect strategies — and explore how these findings can inform teaching and curriculum development in Basic Calculus.

Methodology

Research Design

A descriptive research design was employed to examine and classify the errors and misconceptions respondents exhibited when solving limit problems in calculus. This design was chosen because it enables systematic, in-depth documentation of naturally occurring student work without manipulating variables or conditions (Sandelowski, 2000). It is particularly well-suited to error analysis research, where the goal is to capture, describe, and categorize errors as they appear in authentic academic tasks — providing a faithful picture of students' conceptual and procedural difficulties.

Sources of Data

The study was conducted among Grade 11 Senior High School students enrolled in the Science, Technology, Engineering, and Mathematics (STEM) strand at a higher educational institution in La Union, Philippines, taking Basic Calculus. A total enumeration approach was used, yielding 25 respondents — the entire available cohort for the course during the study period. All participants were informed about the study's purpose and voluntarily provided consent before data collection began. The participants had completed prior mathematics

coursework including Pre-Calculus, which introduced them to functions and their properties, providing the prerequisite conceptual background for limit-related topics. To protect respondents' privacy, all personal identifying information was removed and responses were coded. Anonymity and confidentiality were strictly maintained throughout, in accordance with ethical guidelines for educational research. Respondents were also assured of their right to withdraw at any time without penalty.

Instrumentation and Data Collection

Data were collected using five classroom activities aligned with the Basic Calculus curriculum objectives and adapted from the prescribed Basic Calculus textbook used in the institution. Each activity was designed to assess students' conceptual understanding and procedural fluency in a specific limit topic — including one-sided limits, infinite limits, continuity, and limit laws — ensuring coverage of the major concepts in the curriculum unit. Activities were administered one day after the corresponding topic was formally discussed in class, allowing students to demonstrate their immediate application of newly learned concepts.

Student responses were evaluated using a six-point analytic rubric adopted from the California State Department of Education (1989), *A Question of Thinking*. The rubric was chosen because it provides a structured, criterion-referenced framework for evaluating both conceptual reasoning and procedural accuracy in open-ended mathematics tasks. No modifications were made to the rubric's scoring criteria, though its application was carefully aligned with the specific limit-related learning objectives of this study. Each completed activity was returned to students the following day and discussed with detailed rationale, reinforcing formative feedback as part of the instructional process.

Rubrics:

Point(s) Description

Exemplary response. Gives a complete response with a clear, coherent, Unambiguous and elegant explanation; includes a clear and simplified diagram; communicates effectively to the identified audience; shows understanding of the open-ended problem's mathematical ideas and processes; identifies all the important elements of the problem; may include examples and counterexamples; presents strong supporting arguments.

Competent response. Gives a fairly complete response with reasonably clear explanations; may include an appropriate diagram; communicates effectively to the identified audience; shows understanding of the problem's mathematical ideas and processes; identifies the most important elements of the problem; presents solid supporting arguments.

Minor Flaws But Satisfactory. Completes the problem satisfactorily, but the explanation may be muddled; argumentation may be incomplete; diagram may be inappropriate or unclear; understands the underlying mathematical ideas; uses mathematical ideas effectively.

Serious Flaws But Nearly Satisfactory. Begins the problem appropriately but may fail to complete or may omit significant parts of the problem; may fail to show complete understanding of mathematical ideas and processes; may make major computational errors; may misuse or fail to use mathematical terms; response may reflect an inappropriate strategy for solving the problem.

Begins, But Fails to Complete the Problem. Explanation is not understandable; diagram may be unclear; shows no understanding of the problem situation; may make major computational errors.

Unable to Begin Effectively. Words do not reflect the problem; drawings misrepresents the problem situation; copies parts of the problem but without attempting a solution; fails to indicate which information is appropriate to the problem.

Each activity was returned to the respondents a day after completion and was discussed with a rationale to assess their learning.

Data Analysis Procedures

The five collected activity scripts were systematically reviewed and analyzed in multiple stages. First, the researchers read through all student responses to identify observable errors, marking them for closer examination. Second, each identified error was categorized as either a conceptual understanding error or a mathematical procedural error, based on whether it reflected a failure of knowledge or a failure of execution. Third, ATLAS.ti qualitative data analysis software was used to group and code errors into recurring patterns. Codes were derived inductively from the data, with the researchers independently coding a subset of responses and comparing results to establish inter-rater consistency before finalizing the thematic categories. Disagreements were resolved through discussion until consensus was reached.

For conceptual errors, the percentage of respondents exhibiting each theme per activity was calculated to enable quantitative comparison across activities. Procedural errors were analyzed thematically, with representative examples documented through figures. To provide a richer interpretive layer, Newman's Error Analysis was applied to classify errors by the cognitive stage at which they occurred — reading, comprehension, transformation, process skills, or encoding. The OECD Six-Stage Problem-Solving Framework was then used to contextualize these error types within the broader problem-solving process, from understanding the problem through communicating the solution.

Results and Discussion

This section presents the findings from the five classroom activities, organized into two main parts: (1) Conceptual Understanding Errors and (2) Mathematical Procedural Errors. Before presenting the results, two analytical frameworks are briefly restated for clarity. Newman's Error Analysis classifies errors into five types: Reading Errors (misreading the

problem), Comprehension Errors (misunderstanding meaning), Transformation Errors (selecting the wrong method), Process Skills Errors (executing steps incorrectly), and Encoding Errors (writing an incorrect final answer or notation). The OECD Six-Stage Problem-Solving Framework maps problem-solving progress through: (1) Understanding the Problem, (2) Characterizing the Problem, (3) Representing the Problem, (4) Solving the Problem, (5) Reflecting on the Solution, and (6) Communicating the Solution.

Table 1.
Errors in Conceptual Understanding

	Activity 1	Activity 2	Activity 3	Activity 4	Activity 5	Average
Theme 1: Incomplete Understanding of Definitions	32%	28%	44%	36 %	52%	38.4%
Theme 2: Challenges in Classifying and Recognizing Different Categories	44%	36%	44%	40%	32%	39.2%
Theme 3: Difficulty Handling Complex Problems	52%	48%	32%	40%	32%	40.8%
Theme 4: Misinterpretation and Misuse of Symbols and Notation	28%	20%	28%	20%	20%	23%
Theme 5: Confusion Over Terminology	28%	28%	20%	20%	24%	24%

As show in Table 1, Theme 1 — Incomplete Understanding of Definitions — was the most evenly distributed error across activities, affecting 32% of respondents in Activity 1, 28% in Activity 2, 44% in Activity 3, 36% in Activity 4, and 52% in Activity 5, with an overall average of 38.4%. This theme captures difficulties with core limit-related definitions, including the formal limit definition (epsilon-delta), limit laws (sum, product, and quotient rules), infinite limits, and transcendental functions (exponential, logarithmic, and trigonometric). The relationship between differentiability and continuity was also frequently misunderstood. Under Newman's Error Analysis, these errors represent Comprehension Errors — students encountered the concepts but failed to grasp their meaning. Within the OECD framework, they correspond to Stage 1 (Understanding the Problem), indicating that many respondents could not correctly interpret the mathematical language before attempting a solution.

Theme 2 — Challenges in Classifying and Recognizing Different Categories — was identified in 44% of respondents in Activity 1, 36% in Activity 2, 44% in Activity 3, 40% in Activity 4, and 32% in Activity 5, averaging 39.2%. This theme is closely linked to Theme 1: when students do not fully understand definitions, they are also unable to correctly classify limit scenarios. Respondents struggled to determine whether a function was continuous or discontinuous, when to apply specific limit laws, and whether a given situation involved a finite

limit, infinite limit, one-sided limit, or two-sided limit. These represent Transformation Errors in Newman's framework — students could read and partly comprehend a problem but could not select the appropriate mathematical approach. In the OECD framework, they reflect failures at Stage 1 (Understanding the Problem), where conceptual ambiguity prevents students from even correctly framing the problem before attempting to solve it.

Theme 3 — Difficulty Handling Complex Problems — emerged as the highest-frequency error theme overall, with an average of 40.8%. Occurrence rates were 52% in Activity 1, 48% in Activity 2, 32% in Activity 3, 40% in Activity 4, and 32% in Activity 5. This theme captures situations where students understood simpler limit cases but faltered when problems involved more elaborate expressions. For instance, problems requiring the normal line through the limit definition — especially when the function contained a complicated expression — were frequently answered incorrectly. Students also failed to apply L'Hôpital's Rule appropriately for rational functions, and showed difficulty analyzing removable discontinuities or determining continuity over intervals. These align with Transformation Errors in Newman's framework and correspond to Stage 4 (Solving the Problem) in the OECD framework — respondents understood the problem type but were unable to execute the appropriate solution strategy under increased complexity.

Theme 4 — Misinterpretation and Misuse of Symbols and Notation — was the least frequent conceptual error, averaging 23%, with 28% in Activity 1, 20% in Activity 2, 28% in Activity 3, 20% in Activity 4, and 20% in Activity 5. Despite its lower frequency, this error type has significant implications: incorrect notation use changes the mathematical meaning of a response entirely. Students misused symbols such as $\rightarrow$ (approaches), ∞ (infinity), and δ (delta) when applying limit laws, and made errors in continuity notation — for example, failing to specify the direction when writing one-sided limits. Under Newman's framework, these are Comprehension Errors, and in the OECD framework they correspond to Stage 1 (Understanding the Problem) — students have not internalized the conventional language required to work with limits precisely.

In Theme 5 — Confusion Over Terminology — the error rate was 28% in Activities 1 and 2, 20% in Activities 3 and 4, and 24% in Activity 5, averaging 24%. Students confused or interchanged terms such as approaches to infinity, continuous, infinity, rate of change, slope, tangent, undefined, and normal line. Because limit problems require precise language, terminological confusion directly interferes with problem interpretation and solution planning. In Newman's framework, these represent Reading Errors — occurring at the earliest stage of engagement with the problem. In the OECD framework, they fall under Stage 1 (Understanding the Problem), confirming that a significant proportion of errors begin even before students attempt any calculation.

Beyond conceptual errors, four recurring procedural error themes were identified from the activity scripts. Unlike conceptual errors, which were quantified by percentage, procedural

errors were analyzed thematically and illustrated with representative examples from student work. These themes are interconnected with the conceptual errors above: procedural breakdowns often stem directly from conceptual gaps, reinforcing the importance of addressing both dimensions in instruction.

Theme 1: Misapplication of Mathematical Rules

Respondents frequently struggled to apply mathematical rules correctly, even in cases where they appeared to understand the general approach. A common example was the use of L'Hôpital's Rule without first verifying that the required indeterminate form ($0/0$ or ∞/∞) was present — a conceptual prerequisite that was often overlooked. Errors in algebraic manipulation were also prevalent, particularly in factoring (Figure 1) and rationalization (Figure 2), suggesting that gaps in prerequisite algebra skills compounded students' difficulties in applying calculus techniques. Under Newman's Error Analysis, these are Transformation Errors, and in the OECD framework they reflect failures at Stage 4 (Solving the Problem) — students selected a method but executed it incorrectly due to an incomplete or faulty understanding of the rule's conditions.

Figure 1.

Errors in Factoring

1) $\lim_{x \rightarrow 2} \frac{x^2+1}{x+1} = \lim_{x \rightarrow 2} \frac{2}{2} = \lim_{x \rightarrow 2} \frac{2}{1} = -2$

2) $\lim_{x \rightarrow 3} \frac{x^2-4}{x-6} = \lim_{x \rightarrow 3} \frac{(x+3)(x-2)}{x-3} = \lim_{x \rightarrow 3} \frac{(x+3)}{1} = 6$

Figure 2.

Error in Rationalization

3) $\lim_{x \rightarrow 2} \frac{\sqrt{x}-\sqrt{2}}{x-2} = \lim_{x \rightarrow 2} \frac{\sqrt{x}-\sqrt{2}}{x-2} \cdot \frac{\sqrt{x}+\sqrt{2}}{\sqrt{x}+\sqrt{2}} = \frac{2\sqrt{2}}{0}$

Theme 2: Incomplete Steps

A second major procedural pattern involved students skipping necessary steps and jumping directly to a final answer. This was particularly evident in problems requiring the verification of differentiability — where students failed to compare derivatives from both sides (Figure 3) — and in checking one-sided limits (Figure 4), where intermediate steps were omitted. The consequence was not merely an incorrect answer, but an inability to justify the solution process — a critical skill in mathematical reasoning. According to Newman's framework, incomplete steps represent Comprehension Errors: students did not fully grasp what the problem required them to demonstrate. In the OECD framework, they reflect a breakdown at Stage 4 (Solving the Problem), where logical sequencing and planning are essential.

Figure 3.

Error in Not Verifying Differentiability

5.) decide if f is differentiable at $x=2$

$$f'(x) = \begin{cases} 2x & \text{if } x \leq 2 \\ 2 & \text{if } x > 2 \end{cases}$$

$$f'(2) = 2(2) = 4$$

$$4 \neq 2$$

not differentiable

Figure 4.

Error in Checking One-Sided Limits

Determine whether the limit exist as x approaches 2 for the piecewise function:

$$f(x) = \begin{cases} x^2 - x + 2 & \text{if } x \leq 2 \\ 2x + 1 & \text{if } x > 2 \end{cases}$$

$$\lim_{x \rightarrow 2} f(x) = 2^2 - 2 + 2 = 4$$

The limit exist.

Theme 3: Inconsistent Notation Usage

$$\lim_{x \rightarrow a^-} \text{ or } \lim_{x \rightarrow a^+} \lim_{x \rightarrow 0} \frac{1}{x} = \infty$$

Some respondents demonstrated an understanding of the mathematical concept they were working with, but could not express it correctly using formal notation. In solving one-sided limit problems (Figure 5), students frequently omitted directional symbols such as $\lim_{x \rightarrow a^-}$ or $\lim_{x \rightarrow a^+}$, or used incorrect symbols altogether — fundamentally altering the mathematical meaning of their responses. Misuse of the infinity symbol was also common: for

example, writing $\lim(x \rightarrow 0) 1/x = \infty$ without specifying the direction of approach (Figure 6), which is mathematically imprecise. Per Newman's framework, these are Comprehension Errors. The OECD framework locates them at Stage 2 (Characterizing the Problem), reflecting a limited understanding of how different notations encode distinct mathematical meanings.

Figure 5.

Error in Solving One-Sided Limit

4. Evaluate the one-sided limits (if they exist):

$$f(x) = x + 4$$

$$\lim_{x \rightarrow 4} f(x) = \frac{1}{-4} = \frac{1}{6} \text{ Does not exist}$$

$$\lim_{x \rightarrow 1} g(x) = \frac{1}{1} = \frac{1}{5}$$

The work is marked with a red diagonal line and a red '2' next to the second limit calculation.

Figure 6.

Error in Using the Infinity Symbol

3. Determine the limit as x approaches 0 for the function:

$$\lim_{x \rightarrow 0} = \frac{1}{x}$$

$$\lim_{x \rightarrow 0} = \frac{1}{0}$$

The work is marked with a red diagonal line and a red '2' next to the first limit calculation.

Theme 4: Inappropriate Strategy Selection

Finally, some respondents chose problem-solving strategies that did not match the requirements of the problem. In cases involving limits of rational functions at a point — where direct substitution after factoring was sufficient — some students applied L'Hôpital's Rule without checking whether the indeterminate form conditions were met (Figure 7), while others used unnecessarily complex approaches when simpler, more elegant methods were available (Figure 8). This pattern suggests that students' strategy selection was driven more by familiarity or habit than by careful problem analysis. Under Newman's framework, these are Process Skills Errors. The OECD framework categorizes them under Stage 4 (Solving the Problem), where respondents were familiar with available tools but selected the wrong one for the task at hand.

Figure 7.*Error in Applying L'Hopital's Rule*

Handwritten student work on lined paper. The student starts with the limit: $\lim_{x \rightarrow \infty} \frac{x^3 - 2x + 4}{x^2 + 3x - 6}$. They then write: $= (\text{applying L'Hopital's rule})$. The next line shows the derivative of the numerator over the derivative of the denominator: $\lim_{x \rightarrow \infty} \frac{3x^2 - 2}{2x + 3}$. This is followed by $= \frac{\infty}{\infty}$ and finally $= \infty$. A red checkmark is drawn over the final result, and a red number '2' is written to the right.

Figure 8.*Error in Using an Overly Complex Approach*

Handwritten student work on lined paper. The student is asked to evaluate the limit as x approaches 3 of a rational function. They write: $\lim_{x \rightarrow 3} \frac{x^2 - 9}{x - 3}$. They then calculate: $= \frac{3^2 - 9}{3 - 3} = \frac{9 - 9}{0} = \frac{0}{0}$. A red checkmark is drawn over the final result, and a red number '2' is written to the right. At the bottom, the student writes: "The limit does not exist".

Conclusion

This study set out to examine and classify the errors in conceptual understanding and mathematical procedure made by Grade 11 STEM students when solving limit problems in calculus. Using a descriptive research design, data from 25 respondents were analyzed through Newman's Error Analysis and interpreted within the OECD Six-Stage Problem-Solving Framework — a combined methodological approach that offers a more nuanced and actionable picture of student difficulties than either framework provides alone.

The most significant finding was that errors were not random but fell into identifiable, recurring patterns rooted in foundational conceptual gaps. Among conceptual errors, difficulty handling complex problems was most prevalent (40.8%), followed by challenges in classifying limit categories (39.2%) and incomplete understanding of definitions (38.4%). Misinterpretation of symbols and notation (23%) and confusion over terminology (24%), while less frequent, are particularly consequential because they alter the mathematical meaning of students' responses. On the procedural side, the four identified themes — misapplication of rules, incomplete steps, inconsistent notation, and inappropriate strategy selection — consistently traced back to the same underlying conceptual deficits, confirming the deep interconnection between conceptual understanding and procedural performance.

$$\lim_{x \rightarrow a^-} \text{ or } \lim_{x \rightarrow a^+}$$

Applying Newman's Error Analysis revealed that errors arose at every stage of engagement with limit problems, from initial misreading of terms and notation (Reading and Comprehension Errors) to incorrect strategy selection (Transformation Errors), flawed execution (Process Skills Errors), and imprecise final expression (Encoding Errors). Interpreted through the OECD framework, the majority of errors clustered at Stage 1 (Understanding the Problem) and Stage 4 (Solving the Problem), indicating that students' difficulties begin well before any calculation — in the interpretation of the problem itself — and persist through execution. This dual-framework analysis is a distinctive contribution of this study: by mapping error types to specific problem-solving stages, it provides teachers with a precise diagnostic tool rather than a general sense of where students struggle.

These findings carry clear and actionable implications for classroom practice. Teachers are encouraged to integrate error analysis activities into regular instruction, asking students to examine and explain worked examples that contain deliberate mistakes — an approach that builds both conceptual clarity and metacognitive awareness (Ashlock, 2010). Conceptual understanding should be reinforced through visual representations, contextual examples, and explicit attention to mathematical language and notation, particularly for terms like limit, continuity, approaches, and infinity. Complex problems should be scaffolded — broken into manageable stages — so that students develop confidence and systematic reasoning rather than resorting to ad-hoc strategy selection. Targeted remediation, guided practice, and structured reflection activities can further help students identify their own error patterns and develop more effective problem-solving habits over time.

This study's scope is intentionally focused — 25 Grade 11 STEM students at a single institution — and its findings should be interpreted accordingly. Future research could extend this analysis to larger and more diverse student populations, or replicate the dual-framework approach across other calculus topics such as derivatives and integrals, where analogous error patterns may emerge. Longitudinal studies tracking how error types evolve as students progress through calculus would also be valuable. By continuing to document and analyze students' errors with this level of specificity, the mathematics education community can move closer to instructional designs that genuinely address the cognitive barriers standing between students and calculus mastery.

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